University of Arkansas at little Rock

Department of Systems Engineering

SYEN 3314 Probability and Random Signals - Summer 2009

Instructor: Dr. Nidhal Bouaynaya

Midterm 2 - Monday, June 22, 2009

- This is a closed book exam.
- Calculators are not allowed.
- There are 8 problems on the exam plus one extra credit (or bonus) problem.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and show all relevant work. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.
- Please be neat and box your final answer, we cannot grade what we cannot decipher.

Name

Problem 1

A student's score on a 10-point quiz is equally likely to be an integer between 0 and 10.

1. What is the probability of an A, which requires the student to get a score of 9 or more?
2. What is the probability the student gets an F by getting (strictly) less than 4?

Problem 2

Monitor two consecutive phone calls going through a telephone switching office. Classify each one as a voice call (v) if someone is speaking, or a data call (d) if the call is carrying a modem or a fax signal. Your observation is a sequence of two letters, each being either v or d. For example, two voice calls corresponds to vv. The two calls are independent and the probability that any one of them is a voice call is 0.8. Denote the identity of call i by C_i. If call i is a voice call, then $C_i = v$; otherwise, $C_i = d$. Count the number of voice calls in the two calls you have observed. N_v is the number of voice calls. Consider the three events $N_v = 0, N_v = 1, N_v = 2$. **Determine whether the following pairs of events are independent**:

1. $\{N_v = 2\}$ and $\{N_v \geq 1\}$
2. $\{N_v \geq 1\}$ and $\{C_1 = v\}$
3. $\{C_2 = v\}$ and $\{C_1 = d\}$
4. $\{C_2 = v\}$ and $\{N_v \text{ is even}\}$

Problem 3

Consider a language containing four letters: A,B,C,D.

1. How many three-letter words can you form in this language?
2. How many four-letter words can you form if each letter appears only once in each word.

Problem 4

The random variable X has PMF

$$P_X(x) = \begin{cases} c\,x^2, & x = 1, 2, 3, 4; \\ 0, & \text{otherwise.} \end{cases}$$

1. Find the value of the constant c
2. Find $P[X \in \{u^2 | u = 1, 2, 3, \cdots\}]$
3. Find the probability that X is an even number
4. Find $P[X > 2]$

Problem 5

The random variable X has CDF

$$F_X(x) = \begin{cases} 0, & x < 1; \\ 0.2, & -1 \le x < 0; \\ 0.7, & 0 \le x < 1; \\ 1, & x \ge 1. \end{cases}$$

1. Draw a graph of the CDF
2. Find $P_X(x)$, the PMF of X.

Problem 6

Given the random variable X is Problem 5, let

$$V = g(X) = |X|.$$

1. Find $P_V(v)$, the PMF of V
2. Find $F_V(v)$, the CDF of V
3. Find $E[V]$

Problem 7

In an experiment to monitor two calls, the PMF of N the number of voice calls is

$$P_N(n) = \begin{cases} 0.1, & n = 0; \\ 0.4, & n = 1; \\ 0.5, & n = 2; \\ 0, & \text{otherwise.} \end{cases}$$

Find

1. The expected value $E[N]$
2. The second moment $E[N^2]$
3. The variance $Var[N]$
4. the standard deviation σ_N.

Problem 8

Use the CDF $F_Y(y)$, in Figure 1, to find the following probabilities

1. $P[Y < 1]$
2. $P[Y \leq 1]$
3. $P[Y > 2]$
4. $P[Y \geq 2]$
5. $P[Y = 1]$
6. $P[Y = 3]$

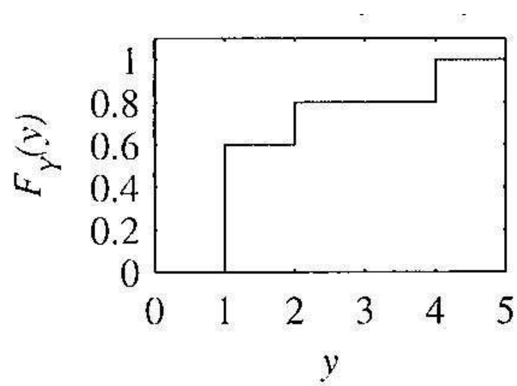

Figure 1: Problem 8

Extra credit problem worth 10 points

Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time.

1. What is $P[-|H]$, the conditional probability that a person tests negative given that the person does have the HIV virus?
2. What is $P[H|+]$, the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive.